Chapter 8
Modeling the Dynamics of Sustainable Peace

Larry S. Liebovitch, Peter T. Coleman, David Futran, Devin Lee, Tamar Lichter, Nicholas Burgess, Daniel Maksumov, and Celine C. Ripla

8.1 Sustaining Peace Is a Lot more than Just Ending War

Peace is not just the absence of war. For decades scholars in conflict resolution have studied the pathologies of war, violence, aggression, and conflict (Deutsch et al. 2014; Deutsch 1977, 2002; Kriesberg 2007; Pruitt et al. 2004). Peace has been studied only in the context of those processes. Very little is known about the fundamental conditions needed to sustain peace. We are therefore studying the complex, multidimensional, and dynamical processes that are needed to sustain peace.

L. S. Liebovitch (✉)
Departments of Physics and Psychology, Queens College, City University of New York, Flushing, NY, USA

Physics Program, The Graduate Center, City University of New York, New York, NY, USA

Adjunct Senior Research Scholar, Advanced Consortium for Cooperation, Conflict, and Complexity (AC4), The Earth Institute at Columbia University, New York, NY, USA
e-mail: larry.liebovitch@qc.cuny.edu

P. T. Coleman
Morton Deutsch International Center for Cooperation and Conflict Resolution (MD-ICCCR), Institute for Psychological Science and Practice (IPSP), Teachers College, Columbia University, New York, NY, USA

Advanced Consortium for Cooperation, Conflict and Complexity (AC4), The Earth Institute at Columbia University, New York, NY, USA

D. Futran · D. Lee · T. Lichter · D. Maksumov
Queens College, City University of New York, New York, NY, USA

N. Burgess
Queens College, City University of New York, New York, NY, USA

Columbia University, New York, NY, USA

C.C. Ripla
Northeastern University, College of Engineering, Boston, MA, USA

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8.2 The Sustaining Peace Mapping Project

In 2014, an interdisciplinary team of scientists, academics, policy-makers, and practitioners convened by the Advanced Consortium on Cooperation, Conflict, and Complexity (AC4) at Columbia University launched a multi-year initiative aimed to provide a more comprehensive and fundamental understanding of sustainable peace. The goals of this project are to:

- Use the scientific evidence from a wide range of disciplines to identify factors that influence sustainable peace
- Create a shared understanding of the relationships between the main factors influencing sustainable peace and their relative importance
- Build on this evidence to create an interactive causal loop diagram to identify effective interventions, measurable goals, and empirically and locally informed indicators for tracking trends in sustainable peace

The core team consists of researchers and practitioners with a broad range of expertise: Peter T. Coleman (social psychology), Joshua Fisher (geography and environmental science), Beth Fisher-Yoshida (communications), Douglas P. Fry (anthropology), Larry S. Liebovitch (physics and psychology), Philippe Vandenbroeck (philosophy), Danny Burns (international development), Kristen Rucki (international education), and Jaclyn Donahue (international development). The core group received extensive additional input from 72 responses on a survey from subject matter experts in a wide range of scientific fields (including neuroscience, evolutionary biology, political science, environmental policy, philosophy) and one small and one larger workshop that included participants from 9 universities in the United States, United Kingdom, and Turkey and representatives from the United Nations, Environmental Law Institute, the United States Institute of Peace, the Inter-American Development Bank, The Omidyar Group, and Bloomberg LP.

A central and evolving product of this work has been to identify the factors needed for sustainable peace in the world and how they influence each other. This is being represented in a visualization called a causal loop diagram. Fig. 8.1 is the current version of the causal loop diagram with the peace factors and their positive (+) and negative (−) influences on each other. The central peace factors PIR and NIR are the positive and negative intergroup reciprocities.

8.3 Mathematical Model of the Causal Loop Diagram of Sustainable Peace

The causal loop diagram is a qualitative description of how these peace factors influence each other. The central goal of the part of the project described here was to develop a rigorous mathematical model of the interactions between these peace factors and use it to determine the dynamics of this system.

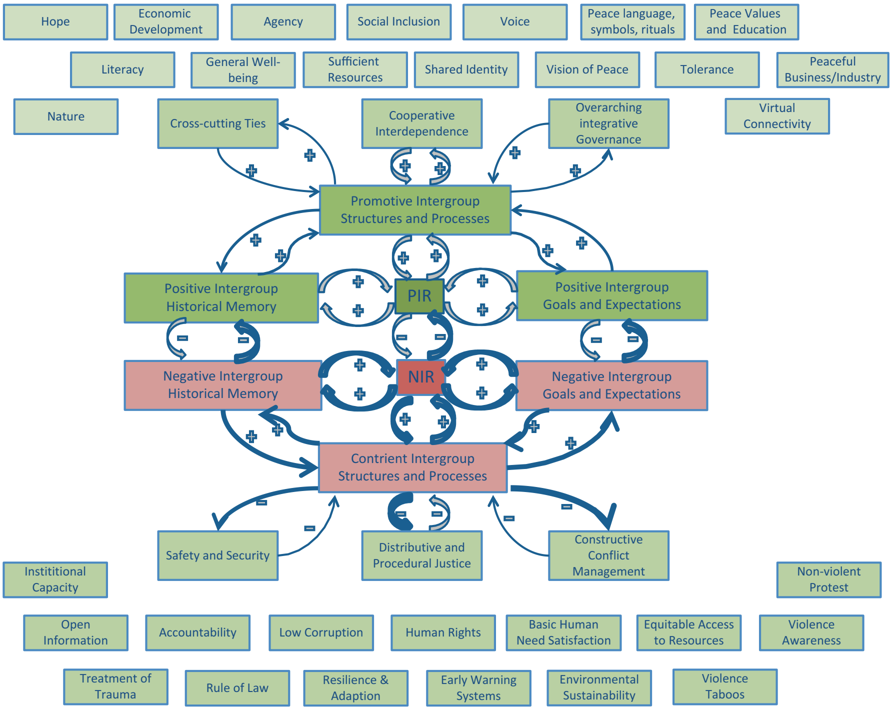


Fig. 8.1 The June 3, 2016, causal loop diagram of sustainable peace from AC4

8.3.1 The Value Added of a Mathematical Model

Transforming the qualitative causal loop diagram into a rigorous mathematical model provides valuable new insights for this project. A mathematical model may be able to:

- Reveal properties about a system that may be difficult to discern in a qualitative causal loop model
- Determine how the quantitative values of the peace factors depend on each other and evolve in time to understand how these peace factors operationally function as a system
- Make quantitative predictions on how the values of the peace factors would change in response to different interventions in the system, such as changing the values of some of the peace factors or the strengths of the influence between them
- Be the input into a graphic display of the quantitative values of the peace factors and the strengths of the connections between them so that researchers can clearly see and explore the effects of changes in the model

8.3.2 Formulation of the Mathematical Model

The quantitative value of each peace factor is given by the variable x_i, and its evolution in time is described by a set of ordinary differential equations:

$$\frac{dx_i}{dt} = -\left|m_i\right| x_i + b_i + \sum_{j=1}^{n} c_{ij} \tanh\left(x_j\right). \tag{8.1}$$

where each term in this equation is motivated by the following considerations:

- dx_i/dt is the rate of change of each variable x_i in time that is determined by the terms on the right-hand side of the equation. We chose to use a first derivative, rather than a higher derivative, to model smoothly varying changes in time.
- $-|m_i|x_i$ is a proportional decay to limit the value of each variable x_i proportionately to its current value, that is, if it is 100, we reduce it to 90, and if it is 1000, we reduce it to 900. We do this to prevent the values of the variables from possibly increasing without bound. We chose a form motivated by the rate of decay of a molecule in a chemical reaction.
- b_i is the self-reinforcement of each variable (such as doing positive intergroup reciprocity makes you feel even better about yourself to do even more positive reciprocity) or an equal input into all the variables from another set of meta-variables at a different level (such as considerations of norms, regulations, institutions, and constraints). We chose a constant value that best represents self-reinforcement for each variable or a constant input from the outside to all the variables.
- c_{ij} is the strength of the influence from variables j to i, that is, from x_j to x_i. We use a hyperbolic tangent function $\tanh(x)$ so that low values of each variable will have a proportionate influence on the other variables, but that influence reaches a threshold of maximum influence when the value of each variable is very high, as illustrated in Fig. 8.2. We chose this functional form as it has been useful in other computational structures, such as the connections between nodes in artificial neural networks.

We have also used similar equations in other models of 2 people interacting and 2000 people interacting (Liebovitch et al. 2008, 2011; Peluso et al. 2012; Fernandez-Rosales et al. 2015) so that we have considerable experience and understanding of both the analytical and numerical behavior of such equations.

8.3.3 Parameters of the Mathematical Model

We choose values for the parameters based on previously published studies and our own experience. For parameters where data is not available, we explore the properties of the model as those parameters are varied over a wide range of values.

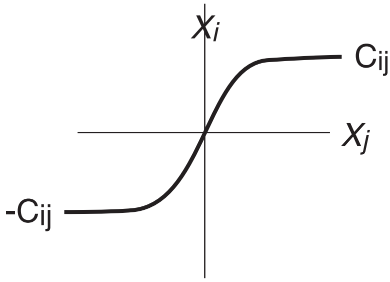


Fig. 8.2 The influence of variable x_j on variable x_i

- c_{ij} is the strength of the influence from variables j to i. This is the most important parameter in transforming the qualitative causal loop diagram into a rigorous mathematical model. In a causal loop diagram, there is no quantitative value for the influence between the variables. In a rigorous mathematical model, the quantitative connection strengths between the variables determine if there is a single or multiple attractors (values of the variables at long times), the type of dynamics of the system (monotonic or oscillatory), and the sensitivity of the dependence on the initial values of the variables. These values will be estimated from published studies, as was done in the preliminary analysis described below.
- m_i is the time constant of exponential decay, which is the degree of memory of the system, and was also called the inertia to change by Gottman et al. (2005). What is most important here is the relative value of this parameter among the different variables. Gottman et al. (2005) found that negative memories have both a stronger influence, and their effects last longer than positive memories. For this reason we set $m_i = -0.2$ for the variables that represent such memories, such as negative historical memory, while we set $m_i = -0.9$ for all the other variables.
- b_i is the self-reinforcement of each variable or an equal input into all the variables from another set of meta-variables at a different level. As data for these parameters is not available, we will explore the effects on the system of a wide range of these values.
- Initial conditions are the initial values given to the variables at the beginning of the computation. We also vary these over a wide range to determine how the dynamics of the system depends on them.

8.3.4 Solution of the Mathematical Model

The equations were integrated numerically to determine the dynamics, the existence of steady states, and their dependence on initial conditions. We have experience in using many different numerical integration methods (fourth-order Runge-Kutta, predictor-corrector methods, and different finite-difference schemes). Here, because of its simplicity and stability, we chose to use the Euler integration with a suitable small $\triangle t$ step size:

$$x_i\left(t+\Delta t\right)=x_i\left(t\right)+\Delta t\left\{-\left|m_i\right|x_i\left(t\right)+b_i+\sum_{j=1}^{n}c_{ij}\tanh\left(x_j\left(t\right)\right)\right\} \tag{8.2}$$

We run the calculation from different initial conditions with different parameters to identify which variables play the most important roles in the dynamical behavior of the system and its long-term steady states (attractors).

8.4 Initial Results from a Mathematical Model of one Segment of the Causal Loop Diagram

We now present an example to demonstrate what additional information can be gained by using such a mathematical model. We constructed a mathematical model of the "core engine," a small but central component, from the April 4, 2016, version of the sustainable peace map.

8.4.1 Parameters of this Initial Mathematical Model

The variables in this model are:

Positive variables:

#1 – Positive historical intergroup memory
#3 – Positive goals and expectations
#5 – Positive intergroup reciprocity

Negative variables:

#2 – Negative historical intergroup memory
#4 – Negative goals and expectations
#5 – Negative intergroup reciprocity

The causal loop diagram is presented in Fig. 8.3:

Next, we determined the qualitative strengths of the connections between the variables. This was done by an extensive literature review to assign strengths of connections between variables as ordinal variables that were none, low, medium, medium-high, and high, as shown in Fig. 8.4. From our previous experience (Liebovitch et al. 2008), we know that given the values of the parameters m_i and b_i, the values of c_{ij} then define different regimes (bifurcations). From that information we can assign representative numerical values to the connection strengths c_{ij} from the ordinal values, as shown in Fig. 8.5.

Fig. 8.3 Segment of the April 4, 2016, causal loop diagram

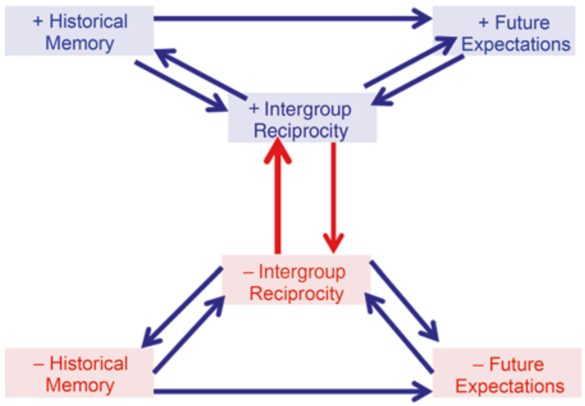


Fig. 8.4 Segment of the April 4, 2016, causal loop diagram with relative ordinal connection strengths

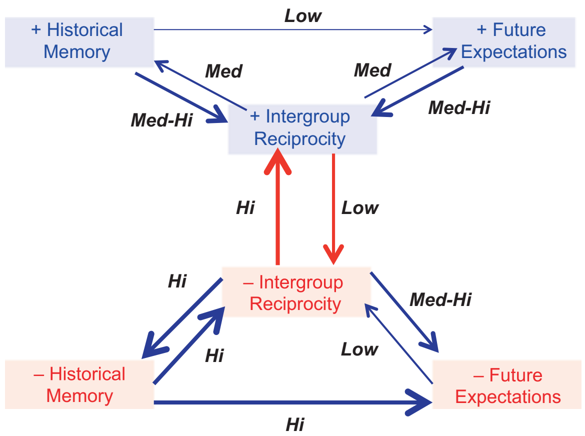
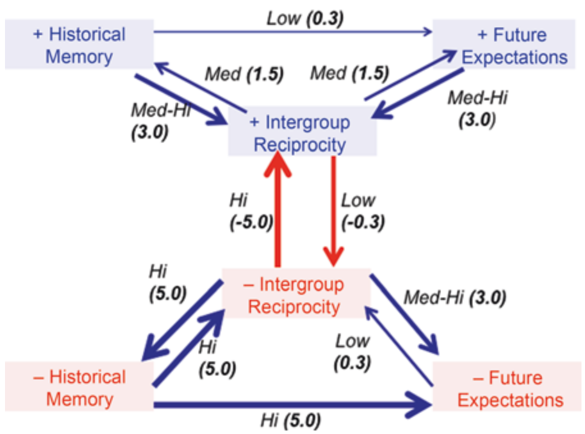


Fig. 8.5 Segment of the April 4, 2016, causal loop diagram with numerical connection strengths



8.4.2 Analysis of this Initial Mathematical Model

We analyzed this model by numerically integrating the equations from a wide range of initial conditions for different sets of parameters. The results of this mathematical model led us to several important conclusions.

Over long times, depending on their initial values, the values of these variables evolve to only one of two possible sets of values, which are called the attractors of this system. There is one "bad" attractor where the values of the negative variables are large and the values of the positive variables are zero and one "good" attractor where the values of the positive variables are large and the values of the negative variables are zero. These long-term attracting values are shown in Table 8.1.

Fig. 8.6 shows the values of the variables as a function of time approaching the "good" attractor. It also shows the strengths of the relative values of each variable at the attractor, as indicated by the size of the circles for each variable, as well as the relative strengths of the connections between the variables defined by the c_{ij} matrix. Figure 8.7 shows the values of the variables as a function of time approaching and at the "bad" attractor.

From the numerical integrations starting with many different initial conditions, we found that the values of the variables *almost always* end up in the bad attractor, at long times. (Only if the initial values of the negative variables are less than 0.01 does this system escape the bad attractor.) Why does this system always go to the bad attractor? The connection strengths of the negative variables are all stronger than those of the positive variables. That means that the negative variables will always reinforce each other and rise to higher values, and then the strongly negative connection from negative intergroup reciprocity to positive intergroup reciprocity forces the positive variables down to zero.

Perhaps this feature should have been recognized in the original causal loop diagram. But, that could not be done because the relative strengths of the connections between the variables are not defined in the original causal loop diagram. Perhaps this feature should have been recognized once the ordinal values of the connection strengths were determined. It was still far from obvious that would be the case. *These findings demonstrate the value of transforming the qualitative causal loop diagram into a quantitative rigorous mathematical model and then using the math-*

Table 8.1 Long-term values of the variables

Variable #	Variable name	Bad attractor	Good attractor
1	+ memory	0	1.7
3	+ expectations	0	2.0
5	+ reciprocity (PIR)	0	6.3
2	- memory	24.9	0
4	- expectations	8.9	0
6	- reciprocity (NIR)	5.9	0

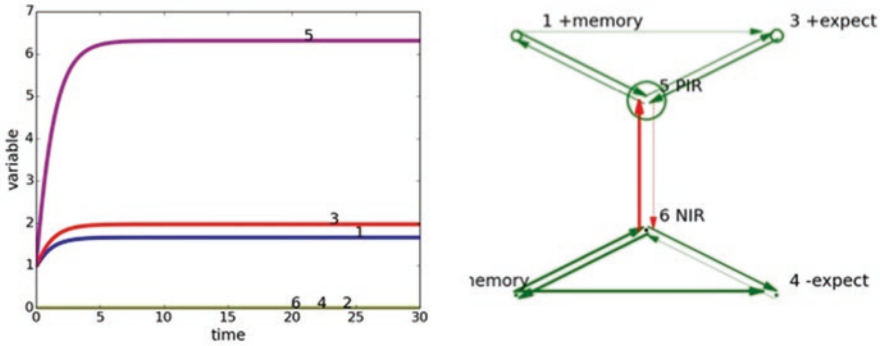


Fig. 8.6 Values of the variables approaching and at the “good” attractor

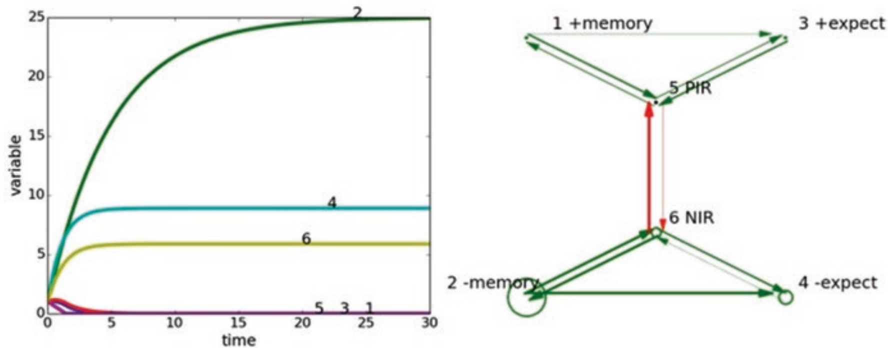


Fig. 8.7 Values of the variables approaching and at the “bad” attractor

ematical and computational methods to analyze the system to discover new and important information in the sustainable peace map.

We don’t believe that it can always be bad. Thus, this analysis tells us that there is an important missing piece or pieces in this model. Additional numerical integrations with different parameter values showed that this system will go to a good attractor if:

- The individual variables reinforce themselves, such as doing good positive intergroup reciprocity makes you feel even better about yourself to do even better positive intergroup reciprocity.
- There is an equal input into all the variables from another set of meta-variables at a different level that represent considerations of norms, regulations, institutions, and constraints.

8.5 Future Directions

8.5.1 Analysis of the Complete Causal Loop Diagram

Our initial analysis was based only on a very small subset of the very much larger causal loop diagram of sustainable peace. A central goal of this project is to now develop and use a rigorous mathematical model to determine properties of this entire system that are not obvious or not possible to determine from the complete causal loop diagrams. This will be done by transforming the complete causal loop diagrams into rigorous mathematical models so that we can use the numerical solutions of the equations to determine the dynamical properties of system, the number of attractors, the initial conditions, the parameter values that lead to each attractor, the stable or unstable dynamics of the evolution of the values of the variables in time, and the sensitivity of the system to its respective parameters and variables.

8.5.2 Graphic Display and Interactive Graphic Interface

The utility of the mathematical model depends on the clarity and usefulness of its human-computer interface so that people can understand the results of the model and interact with it in a meaningful way. We are now developing two types of human-computer interfaces:

- A graphic display to illustrate dynamics of the model; how the values of the variables evolve over time, which can be viewed as both still frame graphs; and a time lapse animation where the value of each variable is indicated by a box whose size and/or color changes as the variable evolves in time
- An interactive point-and-click graphic display so that parameters and variables can be changed while the numerical integration is in progress, in a user-friendly way, by someone without computer programing expertise to enable practitioners and policy-makers to use the model to explore the effects of different possible interventions.

8.5.3 Data Science to Measure the Variables

We are now starting to use modern data science techniques to collect data from databases and social media to determine the quantitative values of the peace variables and to test the validity of the values predicted by the mathematical model. For example, the important variables of the strength of the positive and the negative historical memories can be measured from Facebook, Twitter, and trending Google searches for words that identify “past” or “future,” and then a computer program does a "sentiment analysis" by evaluating the net emotional positivity or negativity of the words in those posts. In our preliminary studies so far:

- We have written scripts in Python and R that "scrape" data from Twitter feeds from specific individuals or within specific geographic areas. We have also used databases of the emotional content of specific words to identify the net positive or negative emotional content in text (Guzmán-Vargas et al. 2015), which will be applied to these tweets.
- Since many of the variables in the causal loop diagram are intergroup variables, we are also exploring ways to identify group membership. For example, to test our programs to define a membership group, we have used Force Atlas 2 and Gephi (Bastian et al. 2009) to construct the network of all the hashtags on Twitter that are linked to #BlackLivesMatter.

8.6 Other Applications of this Mathematical Model

Causal loop diagrams have been a valuable tool to analyze and understand the system properties and dynamics of complex systems. They have been used to analyze a wide variety of systems including political systems, the causes of genocide, the consequences of teenage pregnancy, and the choices in tackling obesity (Burns 2007; Foresight 2016; Ricigliano et al. 2016). The mathematical framework presented here has been helpful in providing new information about the causal loop diagram of sustainable peace. A similar mathematical approach can also have considerable value in analyzing the causal loop diagrams of a wide variety of other systems.

In a complex causal loop diagram, with many positive and negative feedback loops, it can be challenging to trace out the system-wide effects of changing the value of one variable, or a set of variables, or the feedback loops between the variables. The mathematical model developed here makes it possible to unambiguously determine:

- The long-term steady-state values of all the variables in the system
- The dynamics of how all the variables evolve in time, whether they monotonically approach final values or fluctuate periodically or chaotically around sets of values
- The response of the entire system, that is, the values of all of the variables, to changes in the initial values of the variables or the feedback loops between the variables
- The response of the entire system to an intervention that uses inputs from outside the system to hold a given variable, or a set of variables, at a fixed value

Applying this mathematical model to other systems requires:

- Operational definitions of the variables so that they can be measured

- Estimates or measurements of the quantitative values of the strengths of the connections between the variables
- Sufficient computational resources to numerically integrate the equations and determine their dependence on a wide range of initial conditions and system parameters
- A suitably intuitive graphic display of the results of those computations and an interactive human-computer interface so that the analysis can be appreciated and policy interventions explored by non-technologically sophisticated users

The mathematical formulation we have presented here can be viewed as a general computational structure, using (hyperbolic tangent) transfer functions, similar to those used in artificial neural networks, to link together and therefore compute the effects of the variables acting on each other. In that sense, this computational structure shares properties with artificial neural networks and machine learning systems that transform input data (the initial values of the variables and the strengths of the connections between them) into output data (the final values of the variables and their time course in reaching them) and so it should have broad applicability to a wide range of applications.

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